HW 5.4 #1, 3, 5, 7

1. $x + y = 20$ $\quad x =$ one # $\quad y = 20 - x =$ other #

(a) $f(x) =$ sum of the squares

$$f(x) = x^2 + (20-x)^2 = x^2 + 400 - 40x + x^2$$
$$= 2x^2 - 40x + 400$$

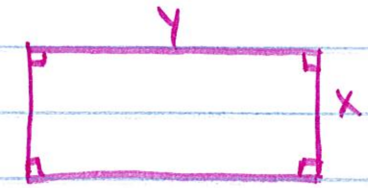
$f'(x) = 4x - 40 \qquad 0 = 4x - 40 \qquad x = 10$

$f(10) = 100 + 100 = 200$

$f(x)$ is a min. of $y = 200$ when $x = 10$ & $y = 10$ since $f'(x)$ changes from Neg. to Pos.

$f(x)$ is the largest when $x = 0$ and $y = 20$ and $f(x)$ would be 400.

3.



$xy = 16\text{ in}^2$ Minimize Perimeter

$y = \dfrac{16}{x} \qquad P = 2x + 2y$

$$P = 2x + 2\left(\frac{16}{x}\right) = 2x + \frac{32}{x}$$

$$P' = 2 + -\frac{32}{x^2} = \frac{2x^2 - 32}{x^2}$$

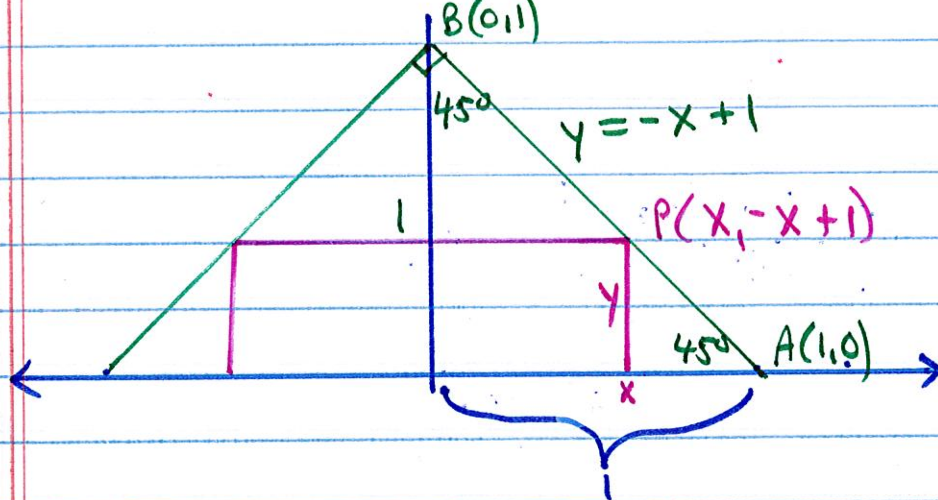
$0 = \dfrac{2x^2 - 32}{x} \qquad 0 = 2x^2 - 32 \qquad 0 = x^2 - 16 \quad x = 4$

If $x = 4$, then $y = 4$

$P(4) = 2(4) + 2(4) = 16$

$P(x)$ is a min. of $P = 16$ in. at $x = 4$ & $y = 4$ since $P'(x)$ changes from Neg. to Pos.

5.4 5.

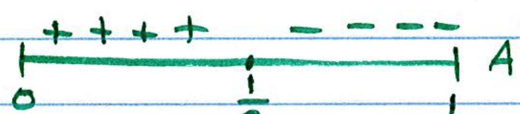


(a) $P(x, ?) = P(x, -x+1)$

(b) Area of rectangle $A(x) = 2x(-x+1) = -2x^2 + 2x$

(c) $A'(x) = -4x + 2 \qquad 0 = -4x + 2 \qquad 4x = 2 \qquad x = \frac{1}{2}$

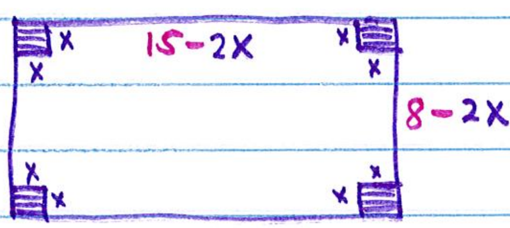
Maximize Area



$A\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{1} = \frac{1}{2}$

$A(x)$ has an abs. max of $A(x) = \frac{1}{2}$ at $x = \frac{1}{2}$ since $A'(x)$ changes from Pos. to Neg.

7.



$V(x) = x(15-2x)(8-2x)$

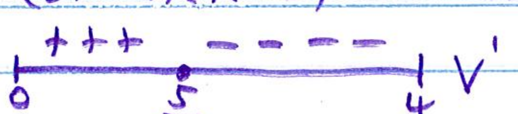
$V(x) = 120x - 46x^2 + 4x^3$

$V'(x) = 120 - 92x + 12x^2$

Maximize Volume $\quad 0 = 120 - 92x + 12x^2$

$0 = 30 - 23x + 3x^2 \qquad 3x^2 - 23x + 30 = 0$

$(3x-5)(x-6) = 0 \qquad x \neq 6 \qquad x = \frac{5}{3} \qquad V\left(\frac{5}{3}\right) = 90.741 \text{ in}^3$



$V(x)$ has an abs max of $V = 90.741$ at $x = \frac{5}{3}$ since V' changes from Pos. to Neg.